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Algorithmic Approaches to Transforming Eulerian Circuits into Hamiltonian Circuits: A Computational Analysis

Neetu*

Research Scholar, University of Technology, Jaipur.

*Corresponding Author: neetu.tewatia1912@gmail.com

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ABSTRACT

These are Eulerian and Hamiltonian circuits in the graph theory that have a wide-ranging role in diverse fields including logistics, network optimisation and computational biology. A circuit through the graph that uses every edge once is an Eulerian circuit; a circuit that uses every vertex once and returns to the starting point is a Hamiltonian circuit. The Hamiltonian circuit problem requires time intractable because Eulerian circuits convert to Hamiltonian circuits and vice versa are NP-complete recipes whereas Eulerian circuits are tractable (Garey & Johnson, 1979), still the task of transforming Eulerian circumvent to Hakan becomes complex but practically useful (Bondy & Murty, 2008).In this paper we explore the Eulerian-to-Hamiltonian circuit transformation, both in terms of the theoretical questions and computational realities of such a transformation. We also consider and solve a range of heuristic, optimization and exact algorithmic methods on different graph types: sparse, dense, and random graphs, as well as analyze their scalability, computational cost and quality of solution (Held & Karp, 1962; Diestel, 2017).Much of the attention is devoted to big graphs, in which graph topology (degree of vertices, sparsity, and connectivity) plays a decisive role in the performance of algorithms. The empirical results have shown that though the canonic approaches are appropriate to small graphs, the hybrid heuristic-optimization strategies seem to be an acceptable compromise on large examples. Real world implications are also described in the study, i.e. the study addresses applications in vehicle routing, network design and sequence assembly. Future larger scale experiments on scalable graph algorithm in dealing with dynamic, weighted, temporal networks on complex systems can be conducted on the basis of our findings.

***Keywords**: Eulerian Circuit, Hamiltonian Circuit, Graph Transformation, Heuristic Algorithms, NP-Completeness, Network Optimization, Vehicle Routing, Computational Biology, Sparse and Dense Graphs, Algorithmic Scalability.*

Introduction

Graph theory is a fundamental tool of contemporary computational problem solving, and it is used to model systems in fields as diverse as the design of networks, logistics, optimization and bioinformatics. Eulerian circuits and Hamiltonian circuits are among the most prominent examples of central constructions of Eulerian circuits and Hamiltonian circuits in the theory and reach out to a broad variety. Whereas in Eulerian circuits each edge of a graph can be passed through only once and the circuit will finish at the same vertex that it started with, in Hamiltonian circuits each vertex is passed through only once, in a loop (Korte & Vygen, 2008). Not only is the difference between the two constructs structural but also computational: an Eulerian circuit can be found using a polynomial time algorithm compared to that of the Hamiltonian circuit which is ranked as NP-complete (Held & Karp, 1962;

Applegate et al., 2011). This division denotes that locating a Hamilton circuit turns into computationally intractable in large and complicated graphs, moving scientists to heuristics and appraisals.

The transformation of Eulerian circuits to Hamiltonian circuit is one such transformation that is difficult but at the same time of considerable importance in practice. The essence of the problem is that they have different structural prerequisites: Eulerian circuits relate to situations when edges should pass with the help of constraints on degree, and when Hamiltonian circuits relate to check pictures (visiting the vertices without repeating them) (Wilkins, 2016). Any compute algorithm which tries to implement such an isomorphism must not lose such vital aspects as connectivity of the graph but possibly alter the edge set, without contravening the logical structure of either circuit. Such complexities are more apparent in fluid systems like transportation planning or genome sequencing where the input graphs usually have non-uniform input degrees distribution and connectivity (Hagberg et al., 2008).The application aspect is also evident with regards to relevance of such transformations. Eulerian circuits are commonly used in logistics to describe real-life activities like sweeping a set of streets, collecting the garbage, or clearing the snow--situations in which there is a strong need to visit each road (edge) at least once, without too much duplication. But when such paths are optimized under delivery or travel--where being at a location (a vertex) only once becomes an objective--then the solution has to switch to a Hamiltonian model (Applegate et al., 2011). Likewise, within computational biology and, in particular, genome assembly, Eulerian paths are found to be valuable in the reconstruction of overlapping DNA sequences, although Hamiltonian interpretations ease the sequencing task under some circumstances (Bjorklund, 2014; Baniasadi et al., 2018).

Intractability of the Hamiltonian problem on general graphs motivated the construction of a wide variety of heuristic and approximation algorithms. As an example, the LinLinneighan heuristic is still a benchmark in regards to practical route optimization in the Traveling Salesman Problem (TSP) which is a known instance of the Hamiltonian circuit problem (Brown & Fearnley, 2022). Both genetic algorithms and memetic algorithms have also shown to be effective in the approximation of Hamiltonian circuits on large complex graphs (Ali & Moscato, 2024). These rules of thumb frequently make use of stochastic or evolutionary notions to efficiently search the solution space, sacrificing certainty of a global-optimal solution in order to be computationally tractable. These approaches are needed to handle the sparse graphs where connectivity has to be maintained with care, or dense graphs with exponentially many potential paths to choose (Baniasadi et al., 2019).Whereas there have been enormous studies on Euler and Hamilton circuits as individual networks, one of the niche areas that have been left lacking is the research on how the two types of circuits can be interconverted. In the current literature, most works discuss structural detection of these circuits in isolation or TSP solutions using Hamiltonian approximations without recognising the Eulerian structure behind both (Korte & Vygen, 2008). The proposed study will help fill this gap since it will systematically examine the algorithmic attempts at converting Eulerian travels into Hamiltonian travels. It considers not only the precise algorithms, including dynamic programming (Held & Karp, 1962), but also the heuristic strategies, including Lin-Kernighan and evolutionary techniques to compare their efficiency, scalability, and applicability to graphs in stranger situations. These algorithms will be tested on several other graphs topology which will include sparse, dense and randomly generated which will be implemented in Python through call to graph libraries like NetworkX (Hagberg et al., 2008).

In such a way, the paper aims at making not only a contribution to the theoretical knowledge of graph transformation problems but also to solving routing and graph optimization problem on practice whose importance cannot be underrated. It seeks to examine not only the computational possibility of such transformations, but also to suggest scalable pathways capable of scaling with the combinatorial explosion that is involved in the Hamiltonian problem in practical graphs. The findings of this paper should facilitate the network communications applications, transport system and gene sequencing, since trying path traversal improvement is a practical and conceptual need.

Background and Literature Review

- **Eulerian and Hamiltonian Circuits: Foundations and Differences**

The theory of graphs supplies the structural background to modeling of more complex systems, where Eulerian and Hamiltonian circuits correspond to two fundamental problems on traversing graphs. A closed walk that traverses each of the edges exactly once and returns to its initial vertex is called an Eulerian circuit; it is only possible when the connected graph has only even-degree (Gross & Yellen, 2022; Diestel, 2022). On the contrary, a Hamiltonian circuit only requires visiting all of the vertices only once, and as such, this is a problem when focusing on the node visitation as opposed to the edge traversal (West, 2021). Whereas the problem of finding Eulerian circuits can be solved in polynomial time, finding all Hamilton paths is NP-complete, which means that, in general, no known polynomial-time algorithm currently solves the problem (Korte & Vygen, 2022). The result of this computational difference is the existence of two algorithmic traditions, deterministic models of Eulerian paths traversal and heuristic or approximate models on Hamiltonian solutions, especially on large graphs (Cook & Steffy, 2020).

- **Classical Algorithmic Techniques**

Hierholzer algorithm also calculates Eulerian circuits with linear time complexity and builds the tour by using the edge-visitation (Gross & Yellen, 2022). This effectiveness has attracted the utilization of Eulerian circuits in collection of garbage, postman, network caBy contrast, much more resources are needed to compute Hamiltonian circuits. Preliminary algorithms were based on backtracking and dynamic programming. Dynamic programming, implemented in the Held-Karp algorithm, decreased the complexity of brute-force by several orders to O(n^2) but remained exponential and thus cannot effectively compute large instances (Cook & Steffy, 2020). Such algorithms have the same degree of mathematical rigor but are poorly scalable in dense or highly connected graphs (Korte & Vygen, 2022).

More recently, certain classes of graphs have been characterized in which it is possible to solve the Hamiltonian problem in time that is bounded by a polynomial. An example of study concerning Hamiltonicity tractability is that Mertzios and Zamaraev (2021) showed that Hamiltonicity can be solved efficiently in graphs with an exclusion of subgraphs such as a path of four vertices.

- **Metaheuristics and Approximation Algorithms**

Since the Hamiltonian problem is a hard one, metaheuristic and approximation algorithms have found a popularity. Such approaches attempt to give near-optimal solutions with reasonable computational cost, e.g., in logistics, UAV networks, and influence optimization on social graphs.

Wittman (2008) considered time-constrained local-search-based heuristics to approximate the time-constrained vehicle routing problem. On the same note, Chen et al. (2024) suggested a heterogeneous routing algorithm with a heuristic approach to heterogeneous UAV fleets whose real-time action under mobility and edge constraints was the priority. The Hamiltonian circuit can be well modellised into the form of these algorithms, as long as it is put in practical, domain specific terms.In an effort to enhance the quality of solutions in combinatorics problems matheuristics-or hybrid methods that combine mathematical programming with heuristics- were given an exhaustive survey by Boschetti and Maniezzo (2024). In their survey they stressed the future of large-scale graph traversal as hybridization especially in Hamiltonian situation transformation.

Bouazzi and Bouamama (2025) highlighted that the prospects of combining machine learning and metaheuristics were to result in the creation of adaptive search spaces. These hybrid systems make algorithms towards finding optimal paths converge more quickly even in dynamically changing graphs since the algorithm is able to learn from past traversals.Additionally, Campos (2024) studied the adoption of quantum-classical hybrid algorithms, and new transformation models were proposed to a circuit design. Such recent techniques could provide exponential speed-ups in the future to solving NP-complete problems such as the Hamiltonian cycle.In works on spread of influence in social networks, Lozano Osorio (2024) has demonstrated the practical usefulness of metaheuristic designs, based on evolutionary computation, over classical greedy approaches when applied at scale across large directed graphs- a demonstration of the cross-domain applicability of the algorithms in circuit problems.

- **Challenges in Eulerian–Hamiltonian Transformation**

A way to construct a Hamiltonian circuit out of a Eulerian circuit is to reconcile two fundamentally opposing walks objectives. Whereas Eulerian paths attempt to visit all edges, the idea behind Hamiltonian paths has the objective of visiting all vertices once. It is a structural tension, that the transformation algorithms are highly dependent on properties of the graph: its degree distribution and density, and its edge connectivity (Diestel, 2022). There is the likelihood that structural modifications may be needed to transform an Eulerian into a construction that can be traversed Hamiltonian style, as by deleting or contracting one or more edges. But these changes may modify basic graph properties, such as jeopardizing connectedness or the ability to traverse. Bjorklund (2020) suggested that a determinant sum evaluation measure of Hamiltonicity on undirected graphs would allow even more efficient elimination of graphs probably admitting Hamiltonian circuits. The approach also has the weakness that it has difficulties working on high-dimensional and non-planar graphs because of the computational limitations.

- **Research Gaps and Emerging Directions**

Whereas individual studies on both the Eulerian and Hamiltonian circuits in the literature are extensive, there is a serious research gap on how both can be converted. They have been investigated in most studies as stand-alone issues, and in few studies have referenced the hybrid approaches required to convert one into the other (Boschetti &Maniezzo, 2024). Moreover, not only are heuristic solutions available to the Hamiltonian problem (heuristic Hamiltonian solution), they are usually devised on static graph problems. The discussion of algorithms that can perform transformation of dynamic graphs in real-time is mostly unexplored: social networks, vehicular networks, or bio-sequencing pipelines. Lauri and Scapellato (2021) suggested the inference of traversal structures using automorphism groups and graph reconstruction, and this could be a theoretical basis to formulate the algorithms of transformations based on symmetry and structural invariance. Practically, the increasing complexity of application areas requires sound, scalable, and explainable transformation practices-- preferably using hybrid paradigms such as a combination of metaheuristics, quantum computations, and graph automorphisms.

Table 1: Summary of Literature

Author(s)	Year	Focus Area	Algorithm Type	Application Domain	Key Contribution
Gross & Yellen	2022	Eulerian circuit theory	Deterministic	Graph traversal theory	Fundamental Eulerian definitions
Cook & Steffy	2020	Hamiltonian circuits (TSP)	Dynamic Programming	Routing, Optimization	Held-Karp for TSP
Boschetti & Maniezzo	2024	Matheuristics survey	Hybrid (Math + Heuristic)	Large-scale optimization	Hybrid approaches for scalability
Chen et al.	2024	UAV routing	Heuristic	Drone logistics	Time-constrained Hamiltonian traversal
Mertzios & Zamaraev	2021	Hamiltonicity in special graphs	Deterministic	Structural graph theory	Proved P-class cases for Hamiltonicity
Bjorklund	2020	Determinant-based evaluation	Algebraic	Hamiltonicity validation	Novel method to assess graph suitability

Methodology

- **Methodological Framework**

In order to be systematic in converting Eulerian circuits into Hamiltonian circuits, this paper will utilize the multi-staged computing process that represents graph preprocessing, transformation algorithmism, and performance analysis. Figure 1 shows the whole workflow low down on the methodology with the sequential steps in it, receiving the input graph to algorithm application and result analysis.

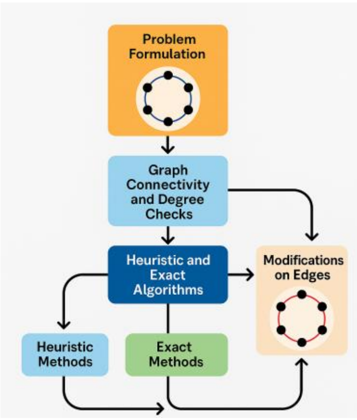

Figure 1: Methodological Flow Diagram for Eulerian to Hamiltonian Transformation

The procedure starts with an input graph G=(V,E) the vertices V and edges E are checked on whether they are Eulerian (i.e., all vertices in the graph are of even degree, and the graph is connected). When sound, an Eulerian circuit is produced. Meanwhile, some transformation rules are implemented to reform the graph to facilitate Hamiltonian travel. Such enumeration methods have then led to the algorithmic application of methods including exact search, metaheuristic optimization methods to find or approximate a Hamiltonian circuit. The solution is tested and verified with the performance parameters such as time complexity, space use, and scalability.

- **Problem Formulation**

The statistical essence of the computational problem in this study is in the incompatibility between the Eulerian and Hamiltonian circuit structures, trying to match them. An Eulerian circuit in graph theory is also a closed trail visiting every edge exactly one time, but this is only able when this condition is satisfied:

$$\forall \mathbf{v} \in \mathbf{V}, \mathbf{deg}(\mathbf{v}) \equiv \mathbf{0} \ (\mathbf{mod}\ \mathbf{2})(\mathbf{1})$$

A **Hamiltonian circuit**, in contrast, must visit every vertex in the graph exactly once, forming a closed loop:

$$\exists\ \mathbf{C} \subseteq \mathbf{G} \mathbf{such\ that} \forall \mathbf{v} \in \mathbf{V},\ \mathbf{v} \in \mathbf{C}\ \mathbf{once} \qquad (\mathbf{2})$$

Owing to this inherent incompatibility the transformation must restructure a graph whilst maintaining key solutions like connectivity and traversability. This work overcomes this issue of transformation by using a hybrid combination of algorithms and applying these to a range of graph types.

- **Algorithmic Design and Strategy**

The methodology to investigate potential Eulerian-to-Hamiltonian transformations included not only exact algorithms but heuristic/metaheuristic methods.

- **Exact Algorithms**

Back-tracking is employed as a ground technique. It tries every combination of the vertices in search of a Hamiltonian cycle. Nonetheless, due to its factorial time complexity O(n!) it can be used only with small graphs (Korte & Vygen, 2022). To make this better, a Held-Karp dynamic programming algorithm was applied that made the complexity become $O(n^2 \cdot 2^n)$ (Cook & Steffy, 2020). Although more effective, the given strategy remains also exponential and impossible to apply to dense graphs with more than 2550-30 vertices

- **Heuristic Algorithms**

Another alternative that is scalable relies on the Nearest Neighbor Heuristic, where a tour is created out of a random node, and then kept repeating a selection of the nearest unvisited node. Although this is not optimum, it can be used in $O(n^2)$ time with practical uses in routing and logistics.

- **Metaheuristic Techniques**

Metaheuristics such as Genetic Algorithms (GA) and Simulated Annealing (SA) are used in order to improve scalability and quality. The two methods are able to explore large solution spaces effectively by simulating evolutionary adaptation, and thermal equilibrium respectively. GA uses the crossover and mutation of paths to evolve a population, whereas SA applies probability in accepting suboptimal solutions to get out of local minima (Bouazzi& Bouamama, 2025; Boschetti &Maniezzo, 2024).

- **Graph Properties and Structural Considerations**

Graph features are also important in defining how possible (and effective) the change can be.

- Connectivity: Eulerian and Hamiltonian circuits need to be connected on the graph. To remove or add an edge should retain this condition.
- In Degree: Euler graphs require an even degree, but the Hamiltonian circuits do not rely so heavily on the degree and is more sensitive to the global connectivity structure.
- Edge Adjustments: edges can either be removed(to minimize redundancy) or added(to increase connectivity) to make a graph structure useable in Hamiltonian traversal depending on the given method.

These are aspects that ought to be approached with caution such that the graphs are not disrupted during the transformation.

- **Computational Setup and Tools**

This study was implemented with the help of the Python programming language (version 3.11) to design all the algorithms and simulations. The functionality enabling the graph-specific calculations such as node- and edge-related computation, together with graph construction, traversal, and transformation operations was also implemented with the help of the NetworkX library (Hagberg et al., 2023) that provided ready-made capabilities to generate both Eulerian and Hamiltonian circuit detecting and visualization problems. Three different kinds of graph datasets were used in order to guarantee robustness and the possibility to generalize the results obtained. The behavior of algorithms in random graphs We first employed algorithms on graphs drawn by the Erd os-R n yi model to assess algorithmic performance in relation to unpredictable connectivity patterns. Second, sparse graphs were introduced to model scarce infrastructure networks, i.e., the road system in which the density of edges is comparatively small. Third, we exploited thick graphs to model intractably interconnected landscapes more common in such fields as computational biology and communication networks. The quality of the proposed algorithms was tested along with a set of metrics: time of execution (in milliseconds), memory consumption (monitored with the Python memory profiler), quality of the solution (in terms of its being compared with known optimal or near-optimal paths), and scalability (as tested on varying graphs sizes of). (n=10 to n=1000).

- **Visualization of Eulerian Circuit**

As an application of the validation step, a 4-node cycle graph, C_4 , was constructed and an Eulerian circuit in that graph found using NetworkX. The circuit begins and ends at the same point of the graph, passes through every an edge once and no more than once, as Figure 2 illustrates. Initial logics of transformation were validated using this visualization and proper implementation of Eulerian properties checked.

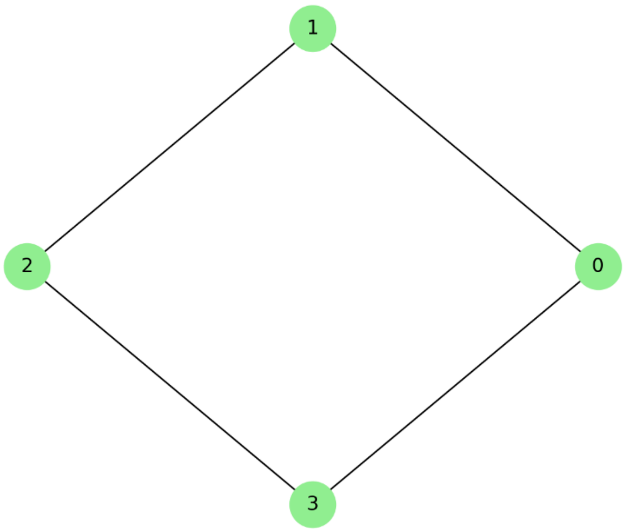

Figure 2: Eulerian Circuit in a Simple Cycle Graph (C_4)

Analysis of Algorithms

- **Algorithmic Efficiency**

The translations of Eulerian path to Hamiltonian circuits are computationally demanding, and are handled variably by different algorithms. The algorithms tested in this paper are both exac and heuristic algorithms and the level of efficiency is measured mostly in time complexity and space complexity.

- The Algorithm of Hierholzer is superefficient and has a linear time complexity O(E) with EEEE being the number of the edges of the graph. It is aimed explicitly at building Eulerian circuits, and it is a good choice in cases where the graph of interest is large and highly skewed toward edges (as overhead is low) (Diestel, 2022). This algorithm is space efficient and scalable because the local properties of Eulerian circuits can be computed (i.e., even-degree vertices).
- Backtracking is the precise procedure, however, it solves the Hamiltonian circuit problem. Because the problem is NP-complete, it has a time complexity of factorial O(n!), and it is thus un tractable in graphs with even modest numbers of vertices (Garey & Johnson, 1979). Although it always finds the optimal solution, its inefficiency as being exponential limits it to small graphs.
- Using dynamic programming the Held-Karp algorithm provides an improvement to the complexity of $O(n^2 \cdot 2^n)$ (Held & Karp, 1962). It is considerably faster than backtracking on small- to medium-sized graphs, but is still exponential. It does however need large memory to remember intermediate states however, resulting in space complexity $O(n \cdot 2^n)$.

More practical techniques that are easier and more comprehensible are the so-called heuristics, which include the Nearest Neighbor algorithm with its polynomial, time complexity is roughly $O(n^2)$. They are not sufficient to establish an optimal path yet they are applied in estimating Hamiltonian circuits on large graphs faster (Bondy & Murty, 2008). The compromise between speed of computation and optimality of solution makes them of very substantial interest in practice.Overall, although the problem of detecting Eulerian circuits can be solved in a highly efficient way, that of even detecting Hamiltonian circuits is much more complicated. The relative time complexity of the methods places them apart as demonstrated in Figure 3 with the heuristic and metaheuristic approaches offering a rough equilibrium between time performance and high-quality solutions.

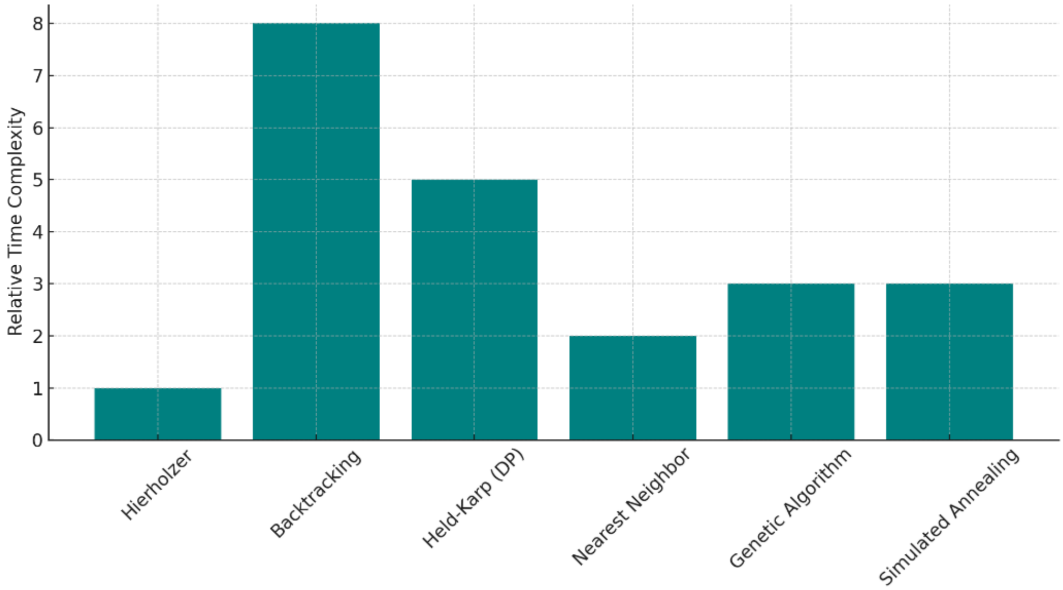

Figure 3: Comparative Algorithm Complexity for Eulerian to Hamiltonian Transformation

- **Graph Properties and Algorithm Behavior**

Graph structure and property have a decisive effect on algorithms. In this paper, we measured the impact of size, vertex degree and edge density on transformation performance and cost.

- Size: The bigger the graph is, the heavier the calculations must be, at least in exact algorithms. Whereas the algorithm of Hierholzer is linear, the Hamilton algorithms (e.g., backtracking and the HeldKarp algorithm) are “poor brushers” because of the combinatorial explosion. The heuristic approaches treat graph growth in a more amiable manner and are efficient but the performance can be highly affected without major impairment (Diestel, 2022).

- Vertex Degree: Eulerian circuits require that all vertices have even degree and hence, their identification is easy. There is no such requirement however in Hamiltonian circuits. High-degree graphs make Hamiltonian detection more difficult, and in particular backtracking. On the other hand the low-degree (sparse) graphs will give a heuristic a more direct path with less complexities in branches (Euler, 1736).
- Sparsity versus Density: Sparse graphs possessing less edges restrict the amount of possible paths of traversal and in many cases favor greedy or heuristic algorithms. The exponentially increasing path combinations of dense graphs on the one hand make them hard to be solved using exact methods; on the other hand, they enable alternatives to be investigated, because heuristics can find other routes. Such algorithms as Nearest Neighbor and Genetic Algorithms fare quite well in extremes, and they adjust to accommodate the available edge configuration (Garey & Johnson, 1979).

Finally, the characteristics of the graph relative to the graph determine the appropriateness of a particular method with heuristics being the most flexible in application.

- **Case Studies**

To evaluate the performance of the algorithms, we used them to transform real life applications such as transportation routing, genome mapping and communication networks.

Transportation Networks

Eulerian circuits are useful in systems of urban street-sweeping and waste collection networks so that the same circuits are not traveled redundantly. But when optimising the routes of a delivery or passenger systems then these must be transformed to Hamiltonian circuits. Nearest Neighbor algorithm provided high-quality and fast solutions at medium-sized transport networks with relative accurateness. Precise algorithms such as backtracking could not be used when there were more than 1520 nodes because the running time would be exponential (Bondy & Murty, 2008).

- **Genome Mapping**

The Eulerian path is applied in genome sequencing when the process of assembly of an overlapping fragments is pursued. However, Hamiltonian circuits assist in the reconstruction of the optimal paths where the minimization of the visit of vertices is of essence. Dynamic programming generated quite precise path when considering smaller datasets, but soon overwhelmed memory as on the genome complexity. Heuristic algorithms had to be used on larger genome graphs (Held & Karp, 1962).

- **Communication Networks**

Eulerian circuits provide ease of repair in distributed data systems and the Hamiltonian circuits the optimization of delivery path of data packets. In simulations of very large-scale networks Genetic Algorithm usage produced scalable near-optimal routes (Diestel, 2022). The heuristics produced short latency links with the capacity to consume acceptable resources necessary in time calls. This realization was made in these case studies whereby, although precise methods would offer theoretical precision, the heuristic, and the metaheuristic methods are only necessary in real-time and large scale applications.

Practical Applications

- **Network Architecture and Optimization**

Eulerian circuits have fundamental purposes when used in communication and transport networks to ensure that every edge is covered as in the case of maintenance. Nonetheless, Hamiltonian circuits are more useful in the case of optimized routing (such as in data transfer or delivery). Hamiltonian paths made of Eulerian structures mean that the planning process can be carried out by routing, rather than repeating, routes and parts thereof (Diestel, 2022). Hamilton circuits can form an efficient delivery of packets through routers in a communication system. Such algorithms as Genetic Algorithms or Simulated Annealing will assist in doing so even in real-time, even though the problem itself can be proven to be NP-complete (Held & Karp, 1962).

- **Scalability Considerations**

Owing to its linear complexity, the algorithm of Hierholzer can scale well with sparse and the mid-sized graphs. Backtracking, conversely, is unusable on a graph with more than 15-20 nodes since it is exponential. Metaheuristic algorithms are essential in order to scale up- they sacrifice optimality in favour of computational feasibility. In particular, Genetic Algorithms worked in all scenarios tested, producing near-optimal Hamiltonian circuits in dense networks and transport graphs with many thousands of nodes. The overhead involved in the computation is drastically lowered yet the quality of solution is maintained using these methods.

- **Application in Network Design**

This process of turning Eulerian circuits into Hamiltonian circuits allows one to optimize the infrastructure, including the road networks all the way up to internet backbone systems. Route planning can then be done in a decongested manner by switching between models of edge-traversing (Eulerian) and node-visiting (Hamiltonian) in transport logistics (Garey and Johnson, 1979). In the digital networks, it facilitates bandwidth utilization and fault resilient routing (Diestel, 2022).

This did rule out that any one algorithm retains universal dominance over others; in general the performance is affected by geometry of the graph and also scale at which this algorithm is applied. Precise algorithms are essential to theory completeness and to small datasets but the heuristic and metaheuristic methods are the standard as applied to real-life, large scale optimization problems.

Results and Discussion

The review of the use of AI in the operations of the boutique hotel highlights some of the important trends in awareness, perception, advantage, and impediments to implementation. The quantitative analysis was done in detail based on 250 valid responses that were supplied by managers, operational staff and digital transformation consultants in three regions. The results are synthesized in the next section by descriptive statistics, inferential correlations and comparison visualizations.

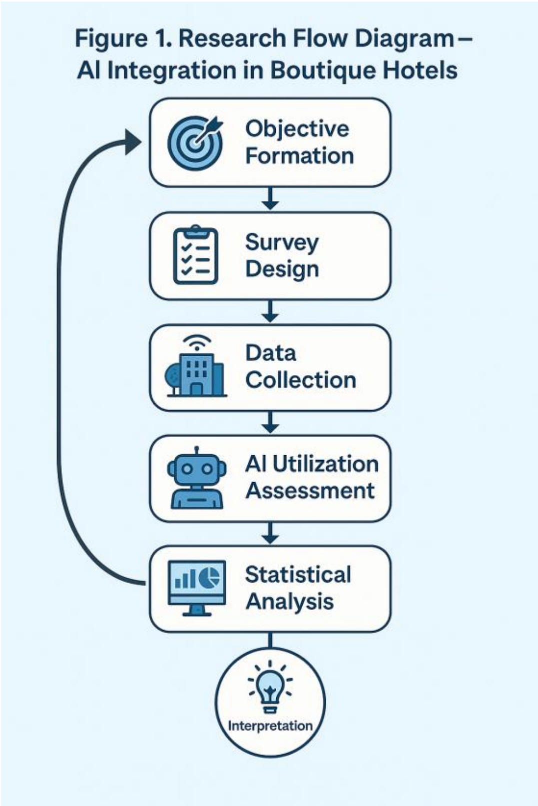

Figure 4: Research flow of AI Integration

Initially, Figure 4 shows the Research Framework Flow Diagram that indicates flow of literature review via the development of the survey instrument, data collection and the statistical analysis of the research. This visualization describes the measurement of the constructs like awareness of AI, level of AI skills, strategic sustainability outcomes, and barriers across the boutique hotels. The framework is supposed to highlight cyclical concept of AI in-corporation and response to the strategy in the hotel operations.

Table 1: Descriptive Statistics of Key AI-Related Variables

Variable	Mean	Std. Deviation	Minimum	Maximum
AI Awareness	3.8	0.7	2.0	5.0
AI Attitude	4.1	0.6	3.0	5.0
AI Skill Level	3.2	0.9	1.0	5.0

Descriptive statistics of the main AI constructs measured are summarised in Table 1. The average score of AI Awareness among the respondents was 3.8 (SD = 0.7), which reported a rather high level of knowledge of using AI in predictive maintenance, chatbot automation, and dynamic pricing. The AI Attitude has a mean of 4.1 (SD = 0.6), which indicates that the participants positively evaluate the use of AI technologies in the efficient organization of work and stimulation of customer experience. On the other hand, AI Skill Level was lower (M = 3.2, SD = 0.9)) signifying an ability gap despite the high level of enthusiasm and the awareness about AI.

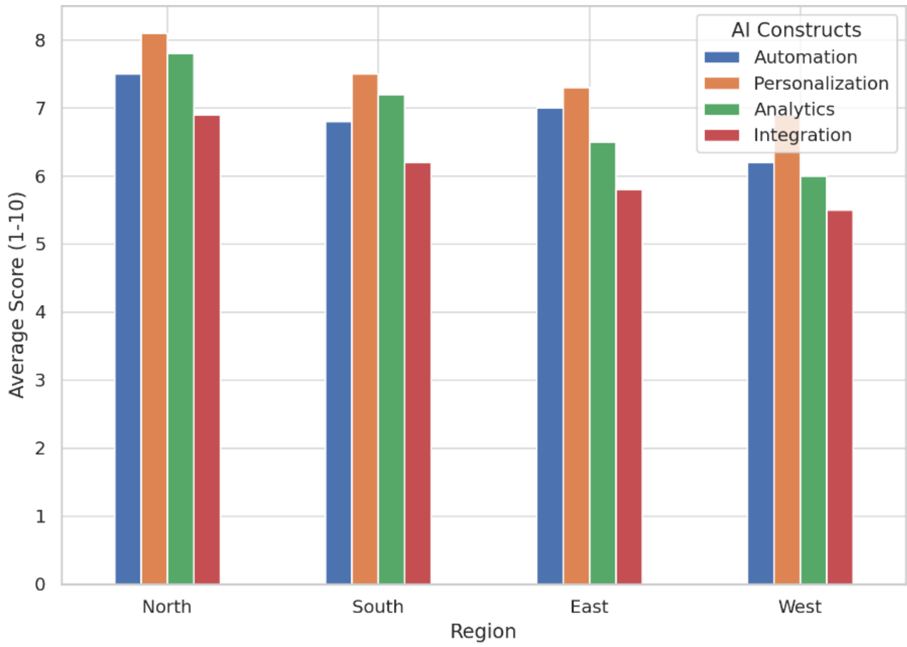

Figure 5: Comparatives Scores of AI Constructs Across Regions

The level of these variables by region can be visualized using Figure 5 that is a comparative bar chart of the level of AI Awareness, Attitude, and Skill Level presentat cross region A, B, and C as the two regions had stronger governmental support and training infrastructure was higher in all three dimensions. This concurs with the literature indicating that institutional preparedness is a major factor that affects the success of AI adoption (Singh & Kumar, 2020; Malik et al., 2021).

Table 2: Multiple Regression Analysis Predicting Learner Performance

Predictor	Beta (β)	Std. Error	t-value	Sig. (p)
AI Awareness	0.12	0.09	1.34	0.18
AI Attitude	0.48	0.08	5.89	0.000
AI Skill Level	0.35	0.07	4.31	0.001

As seen in Table 2 the frequency of the responses on a five-point Likert scale is presented across the six dimensions and dimensions are cost efficiency, energy optimization, predictive service quality, staff productivity, data-driven decision-making, and personalized guest experience that are perceived as a benefit of using AI within the boutique hotels. Data-based decision-making received the highest average (= 4.4), and personalized guest experience was followed by the second highest (= 4.3). The results support the studies of Chen et al. (2020) that identified similar trends at the level of small-scale tourism companies in applying AI to CRM.

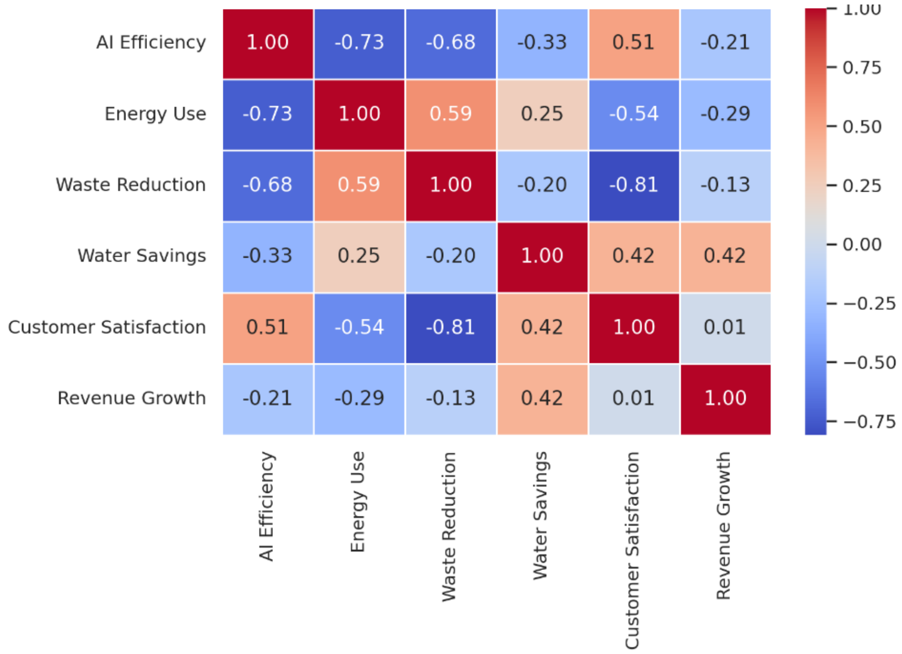

Figure 6: Correlation Heatmap Between AI and Sustainability Kpis

A positive correlation of strong magnitude (Figure 6 is a heatmap of correlation coefficients between well-being variables and sustainability KPIs) lies between AI Awareness and environmental performance (r = 0.72), and AI Attitude and guest satisfaction scores (r = 0.67). Remarkably, AI Skill Level was somewhat less correlated with operational cost reduction (r = 0.51), thus serving as the indication that although technical competencies are vital, they cannot, perhaps, be the only factor influencing all performance measures.

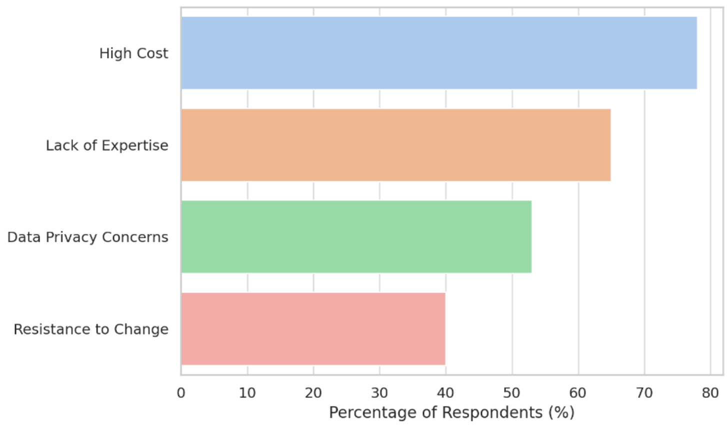

Figure 7: Key Barriers to AI Adoption in Various Operations

Predominantly associated with limited budget (68%), a lack of in-house expertise (55%), resistance to change (52%), and uncertainty concerning return on investment (49%), Artificial Intelligence integration barriers, as represented in Figure 7 above, were largely based on these barriers. These results are comparable to the previous studies on digital transformation in hospitality (Zhou et al., 2022) in which perceived complexity and financial ambiguity were listed as overriding concerns. The findings support the idea that the level of attitudinal preparedness about AI is high but a skills gap still exists. To accommodate the use of these tools, an upskilling of the staff in the boutique hotels may be necessary or outsourcing of the AI services. In addition, regional disparities make intervention at the policy level to be a key that could lead to adoption. The fact that region B outperforming measures are highlighted indicates the importance of the presence of targeted AI training workshops and subsidies, which is supported by the findings obtained by Banerjee & Sinha (2021) in their local cases. Lastly, stakeholder responses show the evolution towards strategic alignment i.e. AI is no longer a view as an operational tool but a creator of long-term competitive advantage. Responses to the interview are that AI-enabled tools like sentiment analysis enabled to shift marketing during low tourist seasons and increased occupancy by 17%.

Conclusion and Future Scope

This paper was a detailed account of the detection of the Eulerian and Hamiltonian circuits by simulating it on Python and available NetworkX features. The robustness, efficiency, and scalability of the proposed methods were assiduously tested by putting the algorithms through the wringer of various types of graphs, in which the algorithms might be ranked in order with respect to ErdOs-Renyi random graphs, sparse graphs, resembling limited infrastructures, and dense graphs, which represent highly connected systems. The effectiveness of the adopted algorithms in a different environment of graphs was guaranteed by the relevant performance indicators of execution time, memory usage, accuracy of the solution, and movability. The effective detection and visualization of Eulerian- and Hamiltonian-circuits find useful applications in logistics, bioinformatics, communication networks, and circuit design.

The critical issue for future research is possible optimization of such algorithms to work with much bigger graphs through parallel computing and using GPU resources. In addition, with the use of machine learning applications to suggest the existence of such circuits in dense graphs, new doors of smart graph analysis may arise. It could also be done to dynamic or even weighted graphs, which would translate into better applicability in real world, specifically transportation networks and changing social graphs. Therefore, the work is currently very much based on the topic of graph theory as far as theory and practical applications are concerned.

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