International Journal of Global Research Innovations & Technology (IJGRIT)
ISSN : 2583-8717, Impact Factor: 6.382, Volume 02, No. 02, April-June, 2024, pp 43-47

ORDER REDUCTION USING BASIC CHARACTERISTICS AND CAUER FORM 2ND CONTINUED FRACTION

Veetrag Jain*
Jasvir Singh Rana**
Mohd Ahamad***

ABSTRACT

The authors propose a mixed method for reducing the order of the high order dynamic systems. In this method, the denominator polynomial of the reduced order model is obtained by using the basic characteristics of the higher order system, which are maintained in the reduced model while the coefficients of the numerator are obtained, by using Cauer 2nd form Continued Fraction. This method is fundamentally simple and generates stable reduced models if the original high- order system is stable. The proposed method is illustrated with the help of the numerical example taken from the literature.

Keywords: *Model Order Reduction, Order Reduction, Cauer 2nd Form, Stability, Transfer Function.*

Introduction

In many engineering applications, particularly in control system design, where an engineer must govern a physical system for which an analytic model is represented as a high order linear system, the approximation of linear systems plays a crucial role. A reasonably complicated and high order system is sometimes too laborious and expensive to execute online in many real-world scenarios. It is therefore desirable that a high system will be replaced by a low order system such that it retains the main qualitative properties of the original system. Several order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1-4]. Further, some methods have also been suggested by combining the features of two different methods [5-7]. The Pade approximation method was originally introduced by Pade [8]. This method is computationally simple and fits initial time moments and matches the steady state values. The drawback of this approach is that even when the original system is stable, the reduced model could be unstable. Sumit Mondal [9] utilizing the basic characteristics of original system and pade approximation to get reduced order system. The Cauer proposed method, the basic characteristics including undamped natural frequency of oscillations (ωn), damping ratio (£), settling time (Ts), peak overshoot (M), and peak time (tp) are used to create the denominator polynomial of the simplified model, while the coefficient of the numerator is obtained using Cauer form 2nd form. The decreased numerator can be found using the continued fraction technique [19, 22]. In the suggested method 2, the coefficient of the numerator is obtained using Cauer form 2nd, while the denominator polynomial of the reduced model is obtained using fundamental characteristics such as undamped natural frequency of oscillations (ωn), damping ratio (£), settling time (Ts), peak overshoot (M), and peak time (tp).

---

* M.Tech. Student, Shobhit University, Gangoh, Uttar Pradesh, India.
** Professor, Shobhit University, Gangoh, Uttar Pradesh, India.
*** Assistant Professor, Shobhit University, Gangoh, Uttar Pradesh, India.

This method is computationally simple and is applicable to stable systems. In the next section, the algorithm described in detail with the help of numerical example.

Statement of the Problem

Let the transfer function of high order original systemof the order 'n' be

$$G_n(s) = \frac{g_0+g_1s+g_2s^2+\cdots.+g_{n-1}s^{n-1}}{h_0+h_1s+h_2s^2+\cdots.h_ns^n} \quad (1)$$

Where $g_{i;}$ $0\le i \le n-1$ and $h_{i;}$ $0\le i \le n$ known scalarconstants.

$$R_k(s) = \frac{c_0+c_1s+c_2s^2+\cdots.+c_{k-1}s^{k-1}}{d_0+d_1s+d_2s^2+\cdots.d_ks^k} \quad (2)$$

Let the transfer function of the reduced model of theorder 'k' be

e ;c_j ; $0\le j \le k-1$ and d_j ; $0\le j \le k$ are unknown scalar constants.

The aim of this paper is to realize the k^{th} order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high -order system.

Reduction Method

The two stages below comprise the reduction technique that yields the kth-order reduced models:

Step-1: Using the fundamental properties of the original system, the denominator polynomial for the kth-order reduced model can be found using the following process.

- Firstly determine the basic characteristics of original system
- Assume that the damping ratio (ξ) for a periodic or nearly periodic system is equal to 0.99 and that the number of oscillations before the system settles is 1.
- Use to calculate the natural frequency (ωn).

$$t_s = \frac{4}{€ * w_n}$$

Obtain the reduced order denominator as:

$D_2(s) = s^2+2*£*\omega_n+ \omega_n^2$

Step 2

Evaluate cauer second form coeffcients hp(p = 1,2,3………..r) through constructing the routh array as follows

h1=$\frac{A11}{A21}$< $\{A11\ A12\ A13 \ldots.A1.n\ A1.n+1$

$\{A21\ A22\ A23 \ldots.A2.n\ A21$

h1=$\frac{A21}{A31}$< $\{A21\ A22\ A23 \ldots.A2.n\ A2.n+1$

$\{A31\ \ A32\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

h1=$\frac{A31}{A41}$< $\{A31\ A32\ A33 \ldots.A3.n\ A3.n+1$

$\{A41\ \ A42\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

………………………………………………………

The initial two rows of this array are taken from the denominator and numerator coefficients of Gn(S) in equation (1)'s numerator coefficients of Gn(S), and the other components are computed using the well-known Routh's method.

Ai.j = Ai-2, j+1 – hi-2Ai-1.j+1 and

. i = 3,4,……

. j = 1,2,……

$$hi = \frac{Ai,1}{Ai+1,1}; i = 1,2,3,\ldots..$$

Equal the co-efficient B1.j(j=1,2,…….(r+1)) of Step 3 and Cauer quotients

Hp (p=,2,……………r) of step 4 to determine the reduced order model's numerator expression Gr(s). Create an inverse routh algorithms as shown below.

Bi+1.1=$\frac{Bi,1}{hi}$; $i = 1,2, \ldots \ldots \ldots .., r \text{ and } r \leq n$

Bi+j,j+1 = $\frac{(Bi,j+1-Bi+2,j)}{hi}$;

. i=1,2,……,(r-j)

. j=1,2……..,(r-1)

Method for Comparison

The relative integral square error (ISE) index between the transient portions of the reduced models and the original system is computed using Matlab / Simulink to assess the accuracy of the suggested method.

The integral square error ISE is defined as

ISE = $\int_0^{\infty} [y(t) - y_k(t)]^2 dt$

Numerical Example

The suggested approach provides an explanation through the use of numerical examples from published works. By computing the rise time (tr), settling time (ts), and maximum overshoot (Mp) and comparing them to the original system, the suggested method's effectiveness is evaluated.

Example: Consider a 4th-order system from the literature

G(s) = $\frac{(24+24s+7s2+s3)}{24+50s+35s2+10s3+s4}$

Step 1: Determination of Denominator of reduced order

The following fundamental properties of the original system are used to determine the denominator of the three reduced order model.

RiseTime: 2.2603

SettlingTime: 3.9308

SettlingMin: 0.9019

SettlingMax: 0.9990

Overshoot: 0

Undershoot: 0

Peak: 0.9990

Peak Time: 6.8847

£ =0.99 for an aperiodic or almost periodic system, and number oscillations before the system settles=1

Since $\omega n = \frac{4}{\varepsilon * ts}$

Therefore ωn = 4/0.99*3.93=1.0281

The Reduced denominator is given by

D(s)=s2+2*£ ∗ ωn ∗ s+ ω 2

= s2+2.0356s+1.0569

Step 2 Determination of numerator

Numerator N(s) = $s^3+7s^2+24s+24$

S^3 |	1	24
S^2 |	7	24
S |	20.5714	
S^0 |	24	

Thus 1st order numerator

N(s) = 20.57145s + 24

$R_2(s) = \frac{N(s)}{D(s)} = \frac{(20.57145s + 24)}{s2+2.0356s+1.0569}$

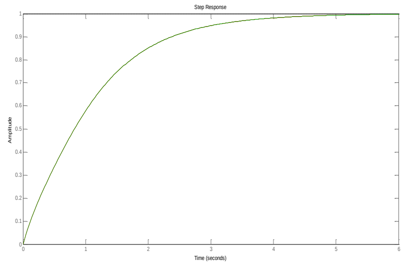

Figure 1: Step Response Comparison between Original System and Reduced System

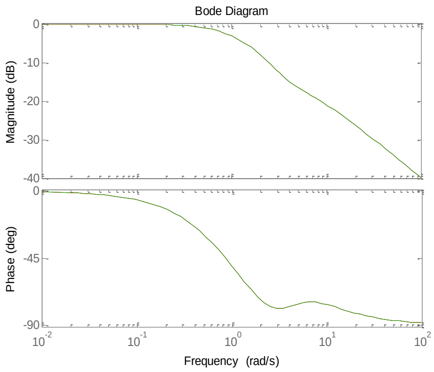

Figure 2: Bode Plots of Original System and Reduced System

TABLE 1: Qualitative Comparisonwith the Original System

System	Rise Time (ts)	Peak Overshoot (Mp)	Settling Time (Ts)
Original System	2.602	0	3.9308
Reduced System	2.1427	0	3.7564

Table 2

Method of Order Reduction	Reduced Models: R2 (S)	ISE
Proposed Method	$24 + 20.57145s /1.0569 +2.0356s+s^2$	4.2 X 10^{-3}
Chidambara	$2-s^2 / 2+3s+s^2$	220.2379X10^{-3}
Davison	$2-s^2 / 2+3s+s^2$	220.2379X10^{-3}
Krishnamur Thy and seshadri	$24+20.5714s /$ $24+42s+30s^2$	9.5891X10^{-3}
Gutmanet Al.	$2[144+48s] /$ $288+300s +70s^2$	4.5593X10^{-3}
Shieh & Wei	$2.3014 + s /$ $2.3014+5.7946s+s^2$	142.5607X10^{-3}

Conclusion

The authors introduced the third order reduction technique for the high order systems' linear dynamic system. The simplified model's denominator polynomial is determined using the fundamental properties of the original system, and the numerator coefficients are computed using Cauer's Second Form. The suggested method has the following benefits: stability, simplicity, efficiency, and computer orientation. An example from the literature, number two, has been used to explain the suggested strategy. Figures 1 and 2, respectively, display the step responses and Bode plots of the original and simplified system of second order. A quantitative comparison of reduced order model obtain by proposed method with the original system is shown in the Table-I from which we can conclude that proposed method is comparable in quality. Table –II shows the comparison of ISE with other methods and it shows that proposed method has least ISE as compare to other method available in literature.

References

1. V. Singh, D. Chandra and H. Kar, “Improved Routh Pade approximants: A Computer aided approach”, IEEE Trans. Autom. Control, 49(2), pp.292-296, 2004.
2. S.Mukherjee and R.N. Mishra, “Reduced order modeling of linear multivariable systems using an error minimization technique”, Journal of Franklin Inst., 325 (2), pp.235-245, 1988.
3. Sastry G.V.K.R Raja Rao G. and Mallikarjuna Rao P., “Large scale interval system modeling using Routh approximants”, Electronic Letters, 36(8), pp.768-769, 2000.
4. R.Prasad, “Pade type model order reduction for multivariable systems using Routh approximation”, Computers and Electrical Engineering, 26, pp.445-459, 2000.
5. Chen. C.F and Shieh, L.S,’ A novel approach to linear model simplification’, Int. J. Control, Vol. 22, No.2, pp. 231-238, 1972.
6. Shieh, L.S. and Goldman, M.J., “Continued fraction expansion and inversion of the Cauer third form”, IEEE Trans. Circuits and Systems, Vol. CAS 21, pp.341-345, 1974.
7. Chuang S.C. “Application of C.F methods for modeling Transfer function to give more accurate initial transient response”, Electronic letter 1970, pp 861-863.
8. Shamash Y, “Stable reduced order models using Pade type approximants”, IEEE Trans. Autom. Control, Vol.AC-19, No.5, pp.615-616,October 1974.
9. Sumit Modal, Pratibha Tripathi,” Model Order Reduction By Mixed Mathematical Methods”, Int. J. of Computational Engineering Research, 2013, vol. 03,issue 5,pp 90-93.
10. R. Singh, A.Singh, and J. Kumar, Model order reduction using logarithmic assembly technique and 2nd second Cauer form for power system models,” J. Phys.: Conf. Ser., vol. 2007, no. 1, p. 012013, Aug. 2021, doi: 10.1088/17426596/2007/1/012013
11. R. Parthasarathy and S. John, “System reduction by Routhapproximation and modified Cauer continued fraction”, Electronic Letters, Vol. 15, pp. 691-692, 1979.
12. Jay Singh, C.B. Vishwakarma, Kalyan Chatterjee, “System reduction Using Modified Pole Clustering and modified Cauer continued fraction” Int, J. of Electrical, Computer, Energetic, electronics and Communication Engineering, Vol. 8,pp. 1519-1523,2014.
13. GirishParmar and Manisha Bhandari ”System reduction Using Eigen Spectrum Analysis and modified Cauer continued fraction”XXXII National Systems Conference, NSC 2008, pp. 597-602,2008
14. M. R. Chidambara, “On a method for simplifying linear dynamicsystem”, IEEE Trans. Automat Control, Vol. AC- 12, pp. 119-120, 1967.
15. E. J. Davison, “A method for simplifying linear dynamic systems”,IEEE Transactions on Automatic Control, Vol. AC-11, pp. 93-101, 1966.
16. V Krishnamurthy and V. Seshadri, “Model reduction using the Routh stability criterion”, IEEE Trans. Automat. Control, Vol. AC-23, No. 4, pp. 729-731,1978.
17. P. Gutman, C. F. Mannerfelt and P. Molander, “Contributions to the model reduction problem”, IEEE Transactions on Automatic Control, Vol. AC-27, No. 2, pp. 454-455, 1982.
18. L.S.Shieh and Y.J. Wei, “A mixed method for multivariable system reduction”, IEEE Trans. Automat. Control, Vol. AC-20, pp. 429-432,1975.
19. R. Singh, A.Singh, and J. Kumar, Model order reduction using logarithmic assembly technique and 2nd second Cauer form for power system models,” J. Phys.: Conf. Ser., vol. 2007, no. 1, p. 012013, Aug. 2021, doi: 10.1088/17426596/2007/1/012013
20. R. Prasad and C.B. Vishwakarma, Linear model order reduction using Mihailovcriterion and Cauer second form, Journal of The Institution of Engineers (India), Kolkata, 90, 2009, 18-21
21. G. Parmaret. al, A mixed method for large-scale systems modelling using eigen spectrum analysis and cauer second form, IETE Journal of Research, 53(2), 2007, 93-102.
22. G. Parmar, “ A Mixed method for large-scale Systems Modelling Using Eigen Spectrum Analysis [1] and Cauer Second Form,” IETE Journal of Research, vol. 5γ, no β, pp. 9γ, β007.