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Analysis of Fractional Partial Differential Equations in Fluid Mechanics via LADM

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ABSTRACT

Fractional calculus extends classical calculus by allowing derivatives and integrals of non-integer order and has important applications in physics, engineering, fluid mechanics, and other fields. Fractional partial differential equations (FPDEs) are widely used to model complex phenomena more accurately than integer-order models. Various analytical and numerical methods have been developed to solve these problems, among which the Laplace Adomian Decomposition Method (LADM) is notable for its simplicity and accuracy. In this study, diffusion equations and FPDEs are solved using LADM with the Caputo operator for fractional derivatives. Results are obtained for both integer and fractional orders and show strong agreement with exact solutions. It is observed that fractional-order solutions converge to integer-order solutions as the order approaches unity. The results also indicate that fractional-order models provide better accuracy compared to classical models. Overall, LADM proves to be an effective and reliable method for solving both linear and nonlinear FPDEs.

***Keywords**: Fractional Partial Differential Equations (FPDEs), Laplace Adomian Decomposition Method (LADM), Fluid Mechanics, Caputo Fractional Derivative, Diffusion Equations.*

Introduction

Fractional calculus is an extension of classical calculus that allows derivatives and integrals of non-integer order. In recent years, it has found significant applications in areas like fluid mechanics, engineering, physics, biology, and signal processing. First used by Abel in 1823 to solve the tautochrone problem, it has developed into a powerful mathematical tool, with fractional partial differential equations (FPDEs) becoming a highly active research field.

Researchers have developed a wide range of analytical and numerical methods to solve fractional differential, integral, and FPDE problems. Techniques such as ADM [11],[12], VIM [10], DTM, homotopy-based methods, and NIM [2] have been widely applied to obtain exact or approximate solutions for various linear and nonlinear equations. These approaches have proven useful in tackling problems like fractional diffusion, Volterra integro differential equations, and boundary value problems, even under complex conditions such as fuzzy initial values. Among these methods, the Laplace Adomian Decomposition Method (LADM) stands out due to its simplicity, accuracy, and efficiency. It provides reliable solutions for both linear and nonlinear ordinary and partial differential equations, including deterministic and stochastic cases. In recent years, LADM has been successfully applied to solve many fluid mechanics problems, highlighting its growing importance in this field.

This paper introduces a straightforward computational method for solving fractional differential equations (FDEs) in one, two, and three-dimensional spaces. The one-dimensional space-time FDE can be written as:

$$\frac{\partial^\alpha}{\partial t^\alpha}v(q,t)=\frac{\partial^2}{\partial q^2}v(q,t)+k(q,t), \quad (1.1)$$

$$c<q<d, 0<t<T', 0<\alpha\leq 1$$

With BC

$$v(q,0)=\emptyset_1(q), v_t(q,0)=\emptyset_2(q),$$
$$v(0,t)=\emptyset_1(t), v_q(0,t)=\emptyset_2(t). \quad (1.2)$$

The 2D space-time fractional differential equation is given by:

$$\frac{\partial^\alpha}{\partial t^\alpha}v(q,r,t)=\frac{\partial^2}{\partial q^2}v(q,r,t)+\frac{\partial^2}{\partial r^2}v(q,r,t)+k(q,r,t), \quad (1.3)$$

$$c<q,r<d, 0<t<T', 0<\alpha\leq 1$$

With BC

$$v(q,r,0)=\varphi_1(q,r), v_t(q,r,0)=\varphi_2(q,r). \quad (1.4)$$

The 3D space-time fractional differential equation is given by:

$$\frac{\partial^\alpha}{\partial t^\alpha}v(q,r,s,t)=\frac{\partial^2}{\partial q^2}v(q,r,s,t)+\frac{\partial^2}{\partial r^2}v(q,r,s,t)+\frac{\partial^2}{\partial s^2}v(q,r,s,t)$$
$$+k(q,r,s,t), \quad (1.5)$$

$$c<q,r,s<d, 0<t<T', 0<\alpha\leq 1$$

$$v(q,r,s,0)=\varphi_1(q,r,s), v_t(q,r,s,0)=\varphi_2(q,r,s). \quad (1.6)$$

Numerical Results

Here, a few standard approaches in fractional calculus—Riemann–Liouville, Grünwald–Letnikov, Caputo, along with the LADM technique—are briefly outlined following [6].

Example 2.1: Let us examine a one-dimensional linear time-fractional differential equation (TFDE).

$$\frac{\partial^\alpha}{\partial t^\alpha}v(q,t)=\frac{\partial^2}{\partial q^2}v(q,t)+2e^q\frac{t^{2-\alpha}}{\Gamma 3-\alpha}-t^2e^q, \quad (2.1.1)$$

$$0<q<1, 0<t<1, 0<\alpha\leq 1.$$

With BC

$$v(q,0)=e^q, v_t(q,0)=-2e^q. \quad (2.1.2)$$

We use equation (2.1.1) and then transform the expression into its Laplace transform (LT) representation.

$$\mathcal{L}\left[\frac{\partial^\alpha v}{\partial t^\alpha}\right]=\mathcal{L}\left[\frac{\partial^2}{\partial q^2}v(q,t)+2e^q\frac{t^{2-\alpha}}{\Gamma 3-\alpha}-t^2e^q\right],$$

$$S^\alpha\mathcal{L}[v(q,t)]-S^{\alpha-1}v(q,0)-S^{\alpha-2}v_t(q,0)=\mathcal{L}[\frac{\partial^2}{\partial q^2}v(q,t)+2e^q\frac{t^{2-\alpha}}{\Gamma 3-\alpha}$$
$$-t^2e^q],$$

$$\mathcal{L}[v(q,t)]=\frac{S^{\alpha-1}v(q,0)}{S^\alpha}+\frac{S^{\alpha-2}v_t(q,0)}{S^\alpha}+\frac{1}{S^\alpha}\mathcal{L}[\frac{\partial^2}{\partial q^2}v(q,t)+2e^q\frac{t^{2-\alpha}}{\Gamma 3-\alpha}$$
$$-t^2e^q]. \quad (2.1.3)$$

Applying the inverse Laplace transform

$$v(q,t)=\mathcal{L}^{-1}\left[\frac{v(q,0)}{S}+\frac{v_t(q,0)}{S^2}+\frac{1}{S^\alpha}\mathcal{L}[\frac{\partial^2 v}{\partial q^2}+2e^q\frac{t^{2-\alpha}}{\Gamma 3-\alpha}-t^2e^q\right]$$
$$=\mathcal{L}^{-1}\left[\frac{v(q,0)}{S}+\frac{v_t(q,0)}{S^2}+\frac{2e^q}{S^3}-\frac{2e^q}{S^{\alpha-3}}+\frac{1}{S^\alpha}\mathcal{L}[\frac{\partial^2 v}{\partial q^2}]\right]. \quad (2.1.4)$$

By applying the ADM procedure, we get.

$$v_0(q,t) \quad = \mathcal{L}^{-1}\left[\frac{v(q,0)}{S} + \frac{v_t(q,0)}{S^2} + \frac{2e^q}{S^3} - \frac{2e^q}{S^{\alpha-3}}\right],$$

$$= \mathcal{L}^{-1}\left[\frac{e^q}{S} - \frac{2e^q}{S^2} + \frac{2e^q}{S^3} - \frac{2e^q}{S^{\alpha-3}}\right],$$

$$v_0(q,t) = e^q - 2te^q + e^q t^2 - \frac{2t^{\alpha+2}e^q}{\Gamma(\alpha+3)}. \quad (2.1.5)$$

$$v_{l+1}(q,t) = \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_l}{\partial q^2}\right]\right], l = 0,1,2,3,\ldots \quad (2.1.6)$$

Setting $l = 0$ in equation (1.6)

$$v_1(q,t) = \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_0}{\partial q^2}\right]\right],$$

$$= \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left[e^q - 2te^q + e^q t^2 - \frac{2t^{\alpha+2}e^q}{\Gamma(\alpha+3)}\right]\right]\right],$$

$$= \frac{e^q t^\alpha}{\Gamma\alpha+1} - \frac{2t^{\alpha+1}e^q}{\Gamma(\alpha+2)} + \frac{t^{\alpha+2}e^q}{\Gamma(\alpha+3)} - \frac{2t^{2\alpha+2}e^q}{\Gamma(2\alpha+3)}. \quad (2.1.7)$$

For l=1

$$v_2(q,t) = \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_1}{\partial q^2}\right]\right],$$

$$= \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left[\frac{e^q t^\alpha}{\Gamma\alpha+1} - \frac{2t^{\alpha+1}e^q}{\Gamma(\alpha+2)} + \frac{t^{\alpha+2}e^q}{\Gamma(\alpha+3)} - \frac{2t^{2\alpha+2}e^q}{\Gamma(2\alpha+3)}\right]\right]\right],$$

$$= \frac{e^q t^{2\alpha}}{\Gamma2\alpha+1} - \frac{2t^{2\alpha+1}e^q}{\Gamma(2\alpha+2)} + \frac{t^{2\alpha+2}e^q}{\Gamma(2\alpha+3)} - \frac{2t^{3\alpha+2}e^q}{\Gamma(3\alpha+3)}, \quad (2.1.8)$$

For l = 2

$$v_3(q,t) = \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_2}{\partial q^2}\right]\right],$$

$$= \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left[\frac{e^q t^{2\alpha}}{\Gamma2\alpha+1} - \frac{2t^{2\alpha+1}e^q}{\Gamma(2\alpha+2)} + \frac{t^{2\alpha+2}e^q}{\Gamma(2\alpha+3)} - \frac{2t^{3\alpha+2}e^q}{\Gamma(3\alpha+3)}\right]\right]\right],$$

$$= \frac{e^q t^{3\alpha}}{\Gamma3\alpha+1} - \frac{2t^{3\alpha+1}e^q}{\Gamma(3\alpha+2)} + \frac{t^{3\alpha+2}e^q}{\Gamma(3\alpha+3)} - \frac{2t^{4\alpha+2}e^q}{\Gamma(4\alpha+3)}. \quad (2.1.9)$$

Therefore, the series representation of the solution is:

$$v(q,t) = e^q[1 - 2t + t^2 - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{t^\alpha}{\Gamma\alpha+1} - \frac{2t^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{t^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{2t^{2\alpha+2}}{\Gamma(2\alpha+3)} + \frac{t^{2\alpha}}{\Gamma2\alpha+1} - \frac{2t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{t^{2\alpha+2}}{\Gamma(2\alpha+3)} - \frac{2t^{3\alpha+2}}{\Gamma(3\alpha+3)} + \frac{t^{3\alpha}}{\Gamma3\alpha+1} - \frac{2t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{t^{3\alpha+2}}{\Gamma(3\alpha+3)} - \frac{2t^{4\alpha+2}}{\Gamma(4\alpha+3)} + \cdots], \quad (2.1.10)$$

At $\alpha = 1$, the LADM solution is given by

$$v(q,t) = e^q[1 - t - \frac{t^2}{2!} + \frac{t^3}{3!} - \frac{t^4}{4!} + \frac{t^5}{5!} - \frac{t^6}{6!} + \cdots] \quad (2.1.11)$$

The exact form of the solution to the above equation is given by

$$v(q,t) = e^q. \quad (2.1.12)$$

Example 2.2 We consider a two-dimensional time-fractional linear diffusion equation.

$$\frac{\partial^\alpha}{\partial t^\alpha}v(q,r,t) = \frac{\partial^2}{\partial q^2}v(q,r,t) + \frac{\partial^2}{\partial r^2}v(q,r,t), \quad (2.2.1)$$

$$0 < q,r < 1, 0 < t < 1, 0 < \alpha \leq 1.$$

With BC

$$v(q,r,0) = e^{q+r}, v_t(q,r,0) = -3e^{q+r}. \quad (2.2.2)$$

We apply the Laplace transform to the equation above

$$\mathcal{L}\left[\frac{\partial^\alpha v}{\partial t^\alpha}\right] = -\mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2}\right],$$

$$S^\alpha \mathcal{L}[v(q,r,t)] - S^{\alpha-1}v(q,r,t) - S^{\alpha-2}v_t(q,r,0) = -\mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2}\right],$$

$$\mathcal{L}[v(q,r,t)] = \frac{S^{\alpha-1}v(q,r,t)}{S^\alpha} + \frac{S^{\alpha-2}v_t(q,r,0)}{S^\alpha} - \frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2}\right]. \quad (2.2.3)$$

Computing the inverse of the Laplace transform

$$v(q,r,t) = \mathcal{L}^{-1}\left[\frac{v(q,r,t)}{S} + \frac{v_t(q,r,0)}{S^2} - \frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2}\right]\right] \quad (2.2.4)$$

Following the ADM procedure, we obtain…

$$v_0(q,r,t) = \mathcal{L}^{-1}\left[\frac{v(q,r,t)}{S} + \frac{v_t(q,r,0)}{S^2}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{e^{q+r}}{S} - \frac{3e^{q+r}}{S^2}\right]$$

$$v_0(q,r,t) = e^{q+r}(1-3t) \quad (2.2.5)$$

Now

$$v_{l+1}(q,r,t) = -\mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_l}{\partial q^2} + \frac{\partial^2 v_l}{\partial r^2}\right]\right], l = 0,1,2,\ldots \quad (2.2.6)$$

For k=0

$$v_1(q,r,t) = -\mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_0}{\partial q^2} + \frac{\partial^2 v_0}{\partial r^2}\right]\right],$$

$$= -\mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left(e^{q+r}(1-3t)\right) + \frac{\partial^2}{\partial r^2}\left(e^{q+r}(1-3t)\right)\right]\right],$$

$$= -2\frac{t^\alpha}{\Gamma\alpha+1}e^{q+r}(1-3t). \quad (2.2.7)$$

For k=1

$$v_2(q,r,t) = -\mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_1}{\partial q^2} + \frac{\partial^2 v_1}{\partial r^2}\right]\right], ,$$

$$= -\mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left(-2\frac{t^\alpha}{\Gamma\alpha+1}e^{q+r}(1-3t)\right) + \frac{\partial^2}{\partial r^2}\left(-2\frac{t^\alpha}{\Gamma\alpha+1}e^{q+r}(1-3t)\right)\right]\right],$$

$$v_2(q,r,t) = 4\frac{t^{2\alpha}}{\Gamma2\alpha+1}e^{q+r}(1-3t). \quad (2.2.8)$$

For k=2

$$v_3(q,r,t) = -\mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_2}{\partial q^2} + \frac{\partial^2 v_2}{\partial r^2}\right]\right], ,$$

$$= -\mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left(4\frac{t^{2\alpha}}{\Gamma2\alpha+1}e^{q+r}(1-3t)\right) + \frac{\partial^2}{\partial r^2}\left(4\frac{t^{2\alpha}}{\Gamma2\alpha+1}e^{q+r}(1-3t)\right)\right]\right],$$

$$v_2(q,r,t) = -8\frac{t^{3\alpha}}{\Gamma3\alpha+1}e^{q+r}(1-3t). \quad (2.2.9)$$

Using LADM, the solution is determined as follows

$$v(q,r,t) = v_0(q,r,t) + v_1(q,r,t) + v_2(q,r,t) + v_3(q,r,t) + \cdots$$

$$v(q,r,t) = e^{q+r}(1-3t)\left[1 - 2\frac{t^\alpha}{\Gamma\alpha+1} + 4\frac{t^{2\alpha}}{\Gamma2\alpha+1} - 8\frac{t^{3\alpha}}{\Gamma3\alpha+1} + \cdots\right] \quad (2.2.10)$$

For $\alpha = 1$,

$$v(q,r,t) = e^{q+r}(1-3t)\left[1 - 2t + \frac{(2t)^2}{2!} - \frac{(2t)^3}{3!} + \cdots\right] \quad (2.2.11)$$

Example 2.3 We consider a 3-dimensional time-fractional linear diffusion equation.

$$\frac{\partial^\alpha}{\partial t^\alpha}v(q,r,s,t) = \frac{\partial^2}{\partial q^2}v(q,r,s,t) + \frac{\partial^2}{\partial r^2}v(q,r,s,t) + \frac{\partial^2}{\partial s^2}v(q,r,s,t), \quad (2.3.1)$$

$$0 < q,r,s < 1, 0 < t < 1, 0 < \alpha \leq 1.$$

With BC

$$v(q,r,s,0) = \sinh(q)\sinh(r)\sinh(s),$$

$$v_t(q,r,s,0) = -\sinh(q)\sinh(r)\sinh(s), \quad (2.3.3)$$

We apply the Laplace transform to the equation above

$$\mathcal{L}\left[\frac{\partial^\alpha v}{\partial t^\alpha}\right] = -\mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial s^2}\right],$$

$$S^\alpha \mathcal{L}[v(q,r,s,t)] - S^{\alpha-1}v(q,r,s,t) - S^{\alpha-2}v_t(q,r,s,0) = \mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial s^2}\right],$$

$$\mathcal{L}[v(q,r,s,t)] = \frac{S^{\alpha-1}v(q,r,s,t)}{S^\alpha} + \frac{S^{\alpha-2}v_t(q,r,s,0)}{S^\alpha} + \frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial s^2}\right]. \quad (2.3.3)$$

Computing the inverse of the Laplace transform

$$v(q,r,s,t) = \mathcal{L}^{-1}\left[\frac{v(q,r,s,t)}{S} + \frac{v_t(q,r,s,0)}{S^\alpha} - \frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v}{\partial q^2} + \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial s^2}\right]\right] \quad (2.3.4)$$

Following the ADM procedure, we obtain…

$$v_0(q,r,s,t) = \mathcal{L}^{-1}\left[\frac{v(q,r,s,t)}{S} + \frac{v_t(q,r,s,0)}{S^2}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{\sinh(q)\sinh(r)\sinh(s)}{S} - \frac{\sinh(q)\sinh(r)\sinh(s)}{S^2}\right]$$

$$= (1-t)\sinh(q)\sinh(r)\sinh(s). \quad (2.3.5)$$

Now

$$v_{l+1}(q,r,s,0) = \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_l}{\partial q^2} + \frac{\partial^2 v_l}{\partial r^2} + \frac{\partial^2 v_l}{\partial s^2}\right]\right], l = 0,1,2,\ldots \quad (2.3.6)$$

For l=0,

$$v_1(q,r,s,0) = \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2 v_0}{\partial q^2} + \frac{\partial^2 v_0}{\partial r^2} + \frac{\partial^2 v_0}{\partial s^2}\right]\right],$$

$$= \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\big((1-t)\sinh(q)\sinh(r)\sinh(s)\big) + \frac{\partial^2}{\partial r^2}\big((1-t)\sinh(q)\sinh(r)\sinh(s)\big) + \frac{\partial^2}{\partial s^2}\big((1-t)\sinh(q)\sinh(r)\sinh(s)\big)\right]\right],$$

$$= -3\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^\alpha}{\Gamma\alpha+1} - \frac{t^{\alpha+1}}{\Gamma\alpha+2}\right) \quad (2.3.7)$$

For l=1,

$$\mathrm{v}_2(\mathrm{q},\mathrm{r},\mathrm{s},0) = \mathcal{L}^{-1}\left[\frac{1}{\mathrm{S}^\alpha}\mathcal{L}\left[\frac{\partial^2 \mathrm{v}_1}{\partial \mathrm{q}^2} + \frac{\partial^2 \mathrm{v}_1}{\partial \mathrm{r}^2} + \frac{\partial^2 \mathrm{v}_1}{\partial \mathrm{s}^2}\right]\right],$$

$$= \mathcal{L}^{-1}\left[\frac{1}{S^\alpha}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left(-3\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^\alpha}{\Gamma\alpha+1} - \frac{t^{\alpha+1}}{\Gamma\alpha+2}\right)\right)\right.\right.$$

$$\left. + \frac{\partial^2}{\partial r^2}\left(-3\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^\alpha}{\Gamma\alpha+1} - \frac{t^{\alpha+1}}{\Gamma\alpha+2}\right)\right)\right.$$

$$+\frac{\partial^2}{\partial s^2}\left(-3\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^{\alpha}}{\Gamma\alpha+1}-\frac{t^{\alpha+1}}{\Gamma\alpha+2}\right)\right)\Bigg]\Bigg],$$

$$= 9\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^{2\alpha}}{\Gamma2\alpha+1}-\frac{t^{2\alpha+1}}{\Gamma2\alpha+2}\right) \quad (2.3.8)$$

$$v_3(q,r,s,0) = \mathcal{L}^{-1}\left[\frac{1}{S^{\alpha}}\mathcal{L}\left[\frac{\partial^2 v_2}{\partial q^2}+\frac{\partial^2 v_2}{\partial r^2}+\frac{\partial^2 v_2}{\partial s^2}\right]\right],$$

$$= \mathcal{L}^{-1}\left[\frac{1}{S^{\alpha}}\mathcal{L}\left[\frac{\partial^2}{\partial q^2}\left(9\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^{2\alpha}}{\Gamma2\alpha+1}-\frac{t^{2\alpha+1}}{\Gamma2\alpha+2}\right)\right)\right.\right.$$

$$+\frac{\partial^2}{\partial r^2}\left(9\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^{2\alpha}}{\Gamma2\alpha+1}-\frac{t^{2\alpha+1}}{\Gamma2\alpha+2}\right)\right)$$

$$+\frac{\partial^2}{\partial s^2}\left(9\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^{2\alpha}}{\Gamma2\alpha+1}-\frac{t^{2\alpha+1}}{\Gamma2\alpha+2}\right)\right)\Bigg]\Bigg],$$

$$= -27\sinh(q)\sinh(r)\sinh(s)\left(\frac{t^{3\alpha}}{\Gamma3\alpha+1}-\frac{t^{3\alpha+1}}{\Gamma3\alpha+2}\right) \quad (2.3.9)$$

$$v(q,r,s,t) = \sinh(q)\sinh(r)\sinh(s)\left[1-t-3\frac{t^{\alpha}}{\Gamma\alpha+1}+3\frac{t^{\alpha+1}}{\Gamma\alpha+2}+9\frac{t^{2\alpha}}{\Gamma2\alpha+1}-9\frac{t^{2\alpha+1}}{\Gamma2\alpha+2}\right.$$

$$\left.-27\frac{t^{3\alpha}}{\Gamma3\alpha+1}+27\frac{t^{3\alpha}}{\Gamma3\alpha+2}+\cdots\right]$$

$$for\ \alpha = 1$$

$$v(q,r,s,t) = \sinh(q)\sinh(r)\sinh(s)\left[1-4t+\frac{12t^2}{2!}-\frac{36t^3}{3!}+\frac{108t^3}{4!}-\cdots\right] \quad (2.3.10)$$

Conclusion

The diffusion equations and fractional partial differential equations (FPDEs) were systematically solved using the Laplace Adomian Decomposition Method (LADM). Fractional derivatives were handled using the Caputo operator. For all examples, solutions were obtained for both integer and fractional orders through LADM. The results demonstrate a strong agreement with the exact solutions for the given cases. In several examples, LADM proved to be a valid and reliable approach. It was also observed that as the fractional order approaches the corresponding integer order, the fractional solutions converge to the exact solutions. Compared to integer-order models, fractional-order models provide a better fit to experimental data when applying LADM. Overall, LADM is shown to be an effective method due to its high level of accuracy. Furthermore, this technique can be extended to solve nonlinear FPDEs.

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